Application of mathematical methods for condition monitoring of oil and gas facilities

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Abstract

Contaminated areas of anthropogenic origin, particularly surface and underground waters as well as soils, have been analyzed. The mathematical models of diffusion processes, based on the use of two- and three-dimensional diffusion equations with a wide range of boundary and initial conditions, have been suggested. It is determined that exact solutions prohibit studying of the peculiarities of their behavior, depending on the type of boundary conditions; therefore we suggest using numerical solutions. The difference schemes of the method of variable directions for numerical realization of two-dimensional models in different frames, accounting for environmental heterogeneity of matter distribution have been presented. The schemes allow to measure concentration of substances in real objects through building of functions, modeling various boundary conditions. A software complex for their implementation has been devised and the test findings as well as their analysis have been presented. To model processes which parameters depend on three spatial coordinates, we have suggested numerical schemes for implementing three-dimensional models that are absolutely stable and have the second order of accuracy at all spatial coordinates.

Keywords: *anthropogenic factors of impact; diffusion equation; environmental state; mathematical model.*

Introduction

Many authors have researched problems of evaluation of the real technical state of oil and gas facilities and their environmental impact [1–3], but complicated environmental situation in the country requires the development of ecological methods that would allow to determine real technical condition of the facility, provide both emergency situations for various reasons (wear equipment over long operating, conditions change, possible sabotage, etc.) and the consequences of such situations.

Herewith, the mathematical modeling methods acquire great significance. They allow making conclusions about the technical condition of real objects, predicting possible incidents at the sites and their consequences on the basis of limited information.

Mathematical models of processes

The problem to be solved can be divided into three main tasks:

1. Defining the parameters of the stress-strain state of the objects in accordance with the known information about the changes in their spatial configuration – usually such information is data on the surface of the moving points of the body, on the basis of which one can build a mathematical representation [3] of the position vector of any point of the body at control time points. For cylindrical objects (multi-purpose pipes, wells with gas-liquid flow, etc.), the presentation can be written as:

$$\begin{aligned}&\boldsymbol{r}(\mathrm{s},\varphi,\mathrm{r},\mathrm{t})=\boldsymbol{r}_l(\mathrm{s},\varphi,\mathrm{r},\mathrm{t})+\\&+\rho(\mathrm{s},\varphi,\mathrm{r},\mathrm{t})(\cos\omega(\mathrm{s},\varphi,\mathrm{r},\mathrm{t})\boldsymbol{b}_\mathrm{l}+\\&+\sin\omega(s,\varphi,\mathrm{r},\mathrm{t})\boldsymbol{n}_\mathrm{l})+\psi(\mathrm{s},\varphi,\mathrm{r},\mathrm{t})\boldsymbol{\tau}_\mathrm{l}-\frac{2\mathrm{R}}{2}\boldsymbol{n}_l,\end{aligned}\qquad(1)$$

where s, φ, r, t are associated with the curved cylindrical body coordinates respectively along the axis of the body $0<S<L$, by theta $0\le\varphi\le 2\pi$ and the radius of the object $R_{leak}<r\le R_{out}$, L is the length of the investigated object; $\boldsymbol{r}_l$ is the radius vector of a point on the generatrix of the object; $\rho(s,\varphi,r,t)$, $\omega(s,\varphi,r,t)$, $\psi(s,\varphi,r,t)$ are the features, characterizing the studied body movement points respectively in the radial, polar and longitudinal directions; $\boldsymbol{\tau}_l,\boldsymbol{b}_l,\boldsymbol{n}_l$ are the tangent vectors, binormali and normal to the generatrix of the object.

Setting of functions $\rho(s,\varphi,r,t)$ and $\psi(s,\varphi,r,t)$, based on the movement of points on the control surface allows time to calculate strain tensor components:

$$\varepsilon_{ij}(s,\varphi,r,t_k)=\frac{1}{2}(g_{ij}(s,\varphi,r,t_k)-g_{ij}(s,\varphi,r,t_0)),\qquad(2)$$

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for the isotropic body:

$$\sigma_{ij} = \lambda I_1(\varepsilon) g_{ij}(s,\varphi,r,t_0) + 2\mu\,\varepsilon_{ij}(s,\varphi,r,t)\,; \quad (3)$$

$$\lambda = \frac{E}{2(\sigma+1)},\ \mu = \frac{E\sigma}{(1+\sigma)(1-2\sigma)}, \quad (4)$$

for the anisotropic model, relations (3) and (4) shall be replaced by the formula [5]:

$$\sigma_{ij}(s,\varphi,r,t) = \sum_{k,l=1}^{3} C_{ijkl}\,\varepsilon_{kl}(s,\varphi,r,t)\,, \quad (5)$$

where g_{ij} is the tensor matrix components, built on (1) [4], and stress tensor components, when deformations are considered to be elastic, recalculation of these components is done by Hooke's law, $I_1(\varepsilon) = \sum_{i=1}^{3}\sum_{j=1}^{3} \varepsilon_{ij}(s,\varphi,r,t) g_{ij}(s,\varphi,r,t_0)$ is the first strain tensor invariant; λ, μ are Lame material parameters [5], related to the Young's modulus E and Poisson's ratio σ of the material relations, C_{ijkl} is the tensor components of the material elastic modules.

The method is presented and validated in details in [3, 5]. Its peculiarity is that the finding of stress-strain state of the object is done on the basis of certain integrated indicators, such as displacements of the body surface without detailing the causes of these movements. Based on the change of the stress-strain state of the object, it is possible to track down potentially dangerous areas where tensions are taking the critical level, or stress change is such that it can cause loss of object continuity as well as other potentially dangerous environmental effects.

2. Assessment of intensity of fluid leakage in case of seal failure at the facility.

The problem of flow parameters estimation in pipelines and downhole flow is narrowed to the necessity to solve the Navies–Stokes equations [6] in two-dimensional formulation:

$$\begin{cases} u\dfrac{\partial u}{\partial x} + v\dfrac{\partial \upsilon}{\partial y} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + \nu\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}\right) + g_x, \\ u\dfrac{\partial v}{\partial x} + v\dfrac{\partial \upsilon}{\partial y} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial y} + \nu\left(\dfrac{\partial^2 \upsilon}{\partial x^2} + \dfrac{\partial^2 \upsilon}{\partial y^2}\right) + g_y, \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial \upsilon}{\partial y} = 0, \end{cases} \quad (6)$$

provided: $p = p_0 - kx$; k is a differential pressure coefficient;

$$\begin{cases} u\big|_{x=0} = -\dfrac{ky^2}{4\mu_1} + \dfrac{kRy}{2\mu_1}, \\ u\big|_{y=0} = u\big|_{y=2R} = 0, \\ \upsilon\big|_{x=0} = \upsilon\big|_{y=0} = 0, \\ \upsilon\big|_{y=2R} = \begin{cases} 0, x \le x_1, x \ge x_2, \\ \pm\upsilon_{leak}, x \in [x_1, x_2], \end{cases} \end{cases} \quad (7)$$

boundary conditions of (7) type let us take into account both possible fluid leakage and the inflow of liquid.

A numerical method for system solving (6–7) has been developed; its convergence and stability have been proved. Also we have built and implemented numerical algorithm and found optimal parameters of the computational grid. This allows to model velocity field for a given value υ_{leak}, which is not always possible to be determined on practice. Therefore, to solve the system (6), the method (6) is applied, reducing system (6) to the Poisson equation:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 2\rho\left(\frac{\partial u}{\partial x}\frac{\partial \upsilon}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial \upsilon}{\partial x}\right). \quad (8)$$

There are certain methods of system solving (6), however, using the results of [7], we can make a conclusion that:

$$\left|2\rho\left(\frac{\partial u}{\partial x}\frac{\partial \upsilon}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial \upsilon}{\partial x}\right)\right| << 1, \quad (9)$$

and, considering (9), we can suggest the following solution algorithm:

on the first stage equation (8) with boundary conditions for Laplace equation (Dirichlet problem) is solved. Moreover, surface relaxation method is applied [6];

solved system (6) with the condition of (7), and υ_{leak} is determined on the basis of $p(x,y,t)$, found in accordance with the Darcy's law:

$$\begin{cases} u = -\dfrac{k}{\mu_1}\dfrac{\partial p}{\partial x}, \\ \upsilon = -\dfrac{k}{\mu_1}\left(\dfrac{\partial p}{\partial y} + \rho g\right), \end{cases} \quad (10)$$

where k is the permeability of the environment, μ_1 is the dynamic viscosity of the fluid;

on the basis of velocity distribution values, the right side of the equation (8) is resolved. The procedure is being repeated till the solutions coincide.

The peculiarity of the obtained solution is that the boundary conditions (7) change at every step of iterative procedure, so in case of iterative process convergence, it is possible to determine the velocity of fluid leakage from the object under investigation.

3. Determination of the concentration of pollutants in the area of emergency.

To assess the concentration of harmful substances, diffusion equation is solved, for which a two-dimensional field is written as:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(a(x,y,t)\frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(a(x,y,t)\frac{\partial C}{\partial y}\right). \quad (11)$$

Coefficient $a(x,y,t)$ is the diffusion coefficient, which depends on the spatial coordinates x,y and time t. Equation (11) is complemented by boundary and initial conditions [8]:

$$C_0(x,y) = C_0(x,y,0)\,, \quad (12)$$

$$\begin{cases} C\big|_{x=0} = C_1(y,t), \\ C\big|_{x=L_1} = C_2(y,t), \\ C\big|_{y=0} = C_3(x,t), \\ C\big|_{y=L_2} = C_4(x,t). \end{cases} \quad (13)$$

To determine the analytical framework $C_i(x,t)$, $C_j(y,t)$, j=1,2 the results of calculations of substance concentration proportionality on the boundary of the substance leakage are used.

The calculation results

The mathematical models have been implemented as computer algorithms. Figure 1 shows potentially dangerous areas of the investigated object (pipe, well piping) in terms of changes in the stress-strain state and possible seal failure. It allows us to estimate the geometric location of possible defects and their intensity, which is characterized by practical stress zone width.

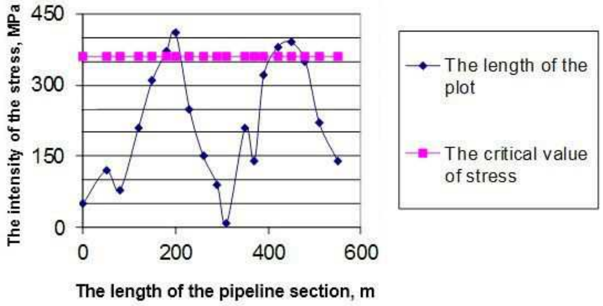

Figure 1 – Distribution of intensity along the length of the pipeline

Figure 2 shows the distribution of pressure in the pipeline system in case of seal failure, depending on the size of breakthrough zone. The maximum product leakage velocity from the system under investigation decreases after the increasing of the hermetic loosing zone sizes. The leakage velocity decreasing is caused by the inner pressure decreasing along the pipeline. The presented model is actual for the leakage of oil products. To model the gas leakage it is necessary to apply the system of gas dynamics equations.

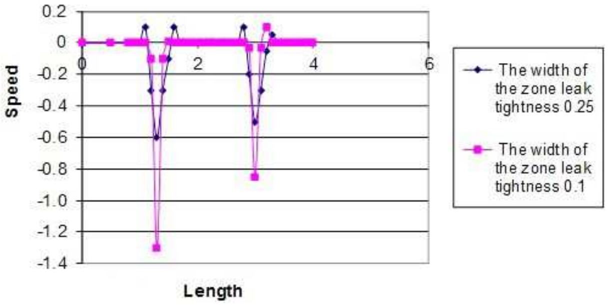

Figure 2 – Average flow velocity components at different values of seal failure zone

Figure 3 shows the distribution of the concentration of harmful substances in the area of object seal failure. The results of velocity components calculations, using (6) – (10), are used to construct the boundary conditions for the concentration (13), that's why the configuration of the concentration distribution zone depends on the leakage velocity value: the concentration distribution zone increases with the increase of the leakage velocity, the same effects can be observed in the process of these value decreasing.

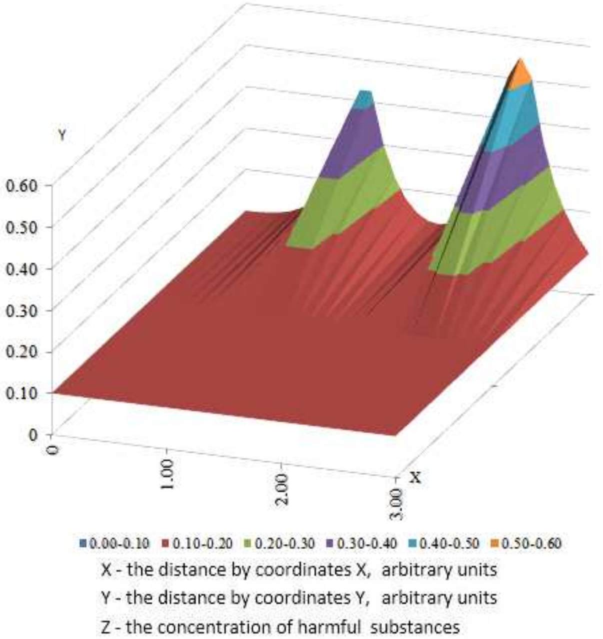

Figure 3 – Average concentration of pollutants in the area of sealing object

The results, presented in Fig. 3 are valid if the diffusion coefficient is constant in the tested zone; in the other cause the configuration of the concentration distribution zone is changed.

Conclusions

The methodology for detection of potentially dangerous areas in terms of changes in the stress-strain state and possible loss of objects geometrics on the basis of the known movements of sets of surface points that allows investigating of both surface and underground sections of pipelines as well as underground borehole column pipes has been suggested.

The mathematical model of viscous fluid flow of in the pipeline system in case of leakage of fluid or filling through the surface, based on numerical integration of systems of Navies-Stokes equations, has been developed.

The method for determination of fluid leakage rate at seal failure, depending on the size of the leakage zone has been identified.

The mathematical model of diffusion of harmful substances has been defined, using models 1, 2 and boundary conditions. All developed numerical models have been implemented.

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Застосування математичних методів до контролю технічного стану об’єктів нафтогазового комплексу

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Проведено аналіз забруднених зон антропогенного походження, зокрема поверхневих, підземних вод та грунтів. Запропоновано математичні моделі дифузійних процесів, що базуються на використанні дво- та тривимірних рівнянь дифузії з широким класом граничних та початкових умов. Встановлено, що точні розв’язки не дозволяють вивчити особливості їх поведінки залежно від типу граничних умов, тому пропонується застосовувати чисельні розв’язки. Запропоновано методику визначення небезпечних з точки зору зміни напружено-деформованого стану ділянок досліджуваних об’єктів шляхом знаходження 6 компонентів тензорів деформацій та напружень, а також методику визначення швидкості витоку продуктів в середовище.

Наведено різницеві схеми методу змінних напрямів для чисельної реалізації двовимірних моделей у різних системах координат з урахуванням неоднорідності середовища поширення речовини. Це дає змогу визначати концентрації цих речовин для реальних об’єктів шляхом побудови функцій, що моделюють різні граничні умови, створено програмний комплекс для їх реалізації, наведено результати тестових розрахунків та їх аналіз. З метою моделювання процесів, параметри яких залежать від трьох просторових координат, запропоновано чисельні схеми реалізації тривимірних моделей, які є абсолютно стійкими та мають другий порядок точності за просторовими координатами.

Ключові слова: *антропогенні чинники впливу, математична модель, рівняння дифузії, стан довкілля.*