SUM OF ENTIRE FUNCTIONS OF BOUNDED L-INDEX IN DIRECTION

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We use definitions and denotations from [1] and [2].

It is known that a product of two entire functions of bounded L-index in direction is a function with the same class (see [2], [4]). But the class of entire functions of bounded index is not closed under addition. The example was constructed by W. Pugh (see [3] and also [5]). Recently we generalized Pugh's example for entire functions of bounded L-index in direction [4].

Meanwhile, there are sufficient conditions of index boundedness for a sum of two entire functions [3].

In this report we present sufficient conditions of boundedness of L-index in direction for a sum of entire functions. They are new for entire functions of bounded l-index too.

We consider an arbitrary hyperplane $A = \{z \in C^n : \langle z, c\rangle = 1\}$, where $\langle c, \mathbf{b}\rangle \neq 0$. Obviously that $\bigcup_{z^0\in A}\{z^0 + t\mathbf{b} : t \in \mathbb{C}\} = \mathbb{C}^n$.

Let $z^0 \in A$ be a given point. If $F(z^0 + t\mathbf{b}) \not\equiv 0$ as a function of variable $t \in \mathbb{C}$ then there exists $t_0 \in \mathbb{C}$ $F(z^0 + t_0\mathbf{b}) \neq 0$. Thus, for every line $\{z^0 + t\mathbf{b} : F(z^0 + t\mathbf{b}) \not\equiv 0\}$ we fixed one point t_0 with specified property. By B we denote a union of those points $z^0 + t_0\mathbf{b}$ i. e. $B = \bigcup_{\substack{z^0\in A\\ F(z^0+t\mathbf{b})\not\equiv 0}} \{z^0 + t_0\mathbf{b}\}$. Clearly that for every $z \in \mathbb{C}^n$ there exist $z^0 \in A$ and $t \in \mathbb{C}$ with property $z = z^0 + t\mathbf{b}$.

Thus, the next theorem is true.

Theorem 1. *Let $L \in Q^n_{\mathbf{b}}$, $\alpha \in (0,1)$ and F, G be the entire in $\mathbb{C}^n$ functions satisfying conditions*

1) *$G(z)$ has bounded L-index in the direction $\mathbf{b} \in \mathbb{C}^n\setminus\{0\}$.*
2) *for every $z = z^0 + t\mathbf{b} \in \mathbb{C}^n$, where $z^0 \in A$, $z^0 + t_0\mathbf{b} \in B$ and $r = |t - t_0|L(z^0 + t\mathbf{b})$ the following inequality is valid*

$$\max\left\{|F(z^0 + t'\mathbf{b})| : \ |t' - t_0| = \frac{2r}{L(z^0 + t\mathbf{b})}\right\} \leqslant$$
$$\leqslant \max\left\{\frac{1}{k!L^k(z^0 + t\mathbf{b})}\left|\frac{\partial^k G(z^0 + t\mathbf{b})}{\partial \mathbf{b}^k}\right| : 0 \leqslant k \leqslant N_{\mathbf{b}}(G_\alpha, L_\alpha)\right\}.$$

3) $c = \sup\limits_{z^0+t_0\mathbf{b}\in B} \dfrac{\max\left\{|F(z^0+t'\mathbf{b})|:\ |t'-t_0|=\frac{2\lambda_2^{\mathbf{b}}(1)}{L(z^0+t_0\mathbf{b})}\right\}}{|F(z^0+t_0\mathbf{b})|} < \infty.$

If $|\varepsilon| \leqslant \frac{1-\alpha}{2c}$ *then the function*

$$H(z) = G(z) + \varepsilon F(z)$$

is of bounded L-index in the direction $\mathbf{b}$ *with* $N_{\mathbf{b}}(H, L) \leqslant N_{\mathbf{b}}(G_\alpha, L_\alpha)$, *where* $G_\alpha(z) = G(z/\alpha)$, $L_\alpha(z) = L(z/\alpha)$.

References

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ON FOURIER QUASICRYSTALS

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A Fourier quasicrystal is a pure point complex measure μ on $\mathbf{R}^p$ such that its Fourier transform in the sense of distributions $\hat{\mu}$ is also a pure point measure. For example, the sum σ of unit masses at the points of $\mathbf{Z}^p \subset \mathbf{R}^p$ is a Fourier quasicrystal, because $\hat{\sigma} = \sigma$ in view of the Poisson summation formula. The support of $\hat{\mu}$ is called spectrum of the Fourier quasicrystal. A set $S \subset \mathbf{R}^p$ is called uniformly discrete if distances between its distinct points are uniformly bounded away from zero. S is called a pure crystal, if it is a finite union of translates of a unique full-rank lattice.

At first we show some new conditions for Fourier transform of measures and distributions to be a measure.

Then we consider a Fourier quasicrystal μ with discrete support Λ. N.Lev, A.Olevskii (2016) proved that if the spectrum of μ and the set of differences $\Lambda - \Lambda$ are both uniformly discrete, then Λ is a subset of a pure crystal.