3) $c = \sup\limits_{z^0+t_0\mathbf{b}\in B} \frac{\max\left\{|F(z^0+t'\mathbf{b})|:\ |t'-t_0| = \frac{2\lambda_2^{\mathbf{b}}(1)}{L(z^0+t_0\mathbf{b})}\right\}}{|F(z^0+t_0\mathbf{b})|} < \infty.$

If $|\varepsilon| \leqslant \frac{1-\alpha}{2c}$ *then the function*

$$H(z) = G(z) + \varepsilon F(z)$$

is of bounded L-index in the direction $\mathbf{b}$ *with* $N_{\mathbf{b}}(H, L) \leqslant N_{\mathbf{b}}(G_\alpha, L_\alpha)$, *where* $G_\alpha(z) = G(z/\alpha)$, $L_\alpha(z) = L(z/\alpha)$.

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ON FOURIER QUASICRYSTALS

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A Fourier quasicrystal is a pure point complex measure μ on $\mathbf{R}^p$ such that its Fourier transform in the sense of distributions $\hat{\mu}$ is also a pure point measure. For example, the sum σ of unit masses at the points of $\mathbf{Z}^p \subset \mathbf{R}^p$ is a Fourier quasicrystal, because $\hat{\sigma} = \sigma$ in view of the Poisson summation formula. The support of $\hat{\mu}$ is called spectrum of the Fourier quasicrystal. A set $S \subset \mathbf{R}^p$ is called uniformly discrete if distances between its distinct points are uniformly bounded away from zero. S is called a pure crystal, if it is a finite union of translates of a unique full-rank lattice.

At first we show some new conditions for Fourier transform of measures and distributions to be a measure.

Then we consider a Fourier quasicrystal μ with discrete support Λ. N.Lev, A.Olevskii (2016) proved that if the spectrum of μ and the set of differences $\Lambda - \Lambda$ are both uniformly discrete, then Λ is a subset of a pure crystal.

We investigate pairs of Fourier quasicrystals μ_1, μ_2 with discrete supports Λ_1, Λ_2 such that the set of differences $\Lambda_1 - \Lambda_2$ is also discrete. We show that the conditions "$\min_j \inf_{x\in\Lambda_j} |\mu_j(x)| > 0$" and "variations of $\hat{\mu}_j$ are uniformly bounded in any ball of radius 1" imply the supports of both measures are subsets of a unique pure crystal. Noté that here we need not the discreteness of spectra of measures. In the case $\Lambda_1 = \Lambda_2$ we get new conditions for support to be a pure crystal.

J.C.Lagarias (2000) conjectured that if μ is a positive measure with a uniformly discrete support and spectrum, then the support of μ is a subset of a pure crystal. The conjecture was proved by N.Lev and A.Olevski (2015). They also proved the corresponding result for any complex measure on the real axis with uniformly discrete support and spectrum.

We construct the singed measure μ on $\mathbf{R}^2$ such that its support and spectrum are uniformly discrete and simultaneously are not subsets of pure crystals. Nevertheless, their supports are unions of two noncommensurable lattices.

The result agrees with A.Cordoba's one (1989). Namely, a uniformly discrete support Λ of any Fourier quasicrystal μ with the property "the set $\{\mu(x) : x \in \Lambda\}$ is finite" is a finite union of translates of several full-rank lattices (maybe noncommensurable).

We replace the above condition by "$F(\mu(x)) = 0$ for all $x \in \Lambda$" for $F(z) = \sum c_{k,m} z^k \bar{z}^m$ to be any convergent series with $c_{0,0} \neq 0$.

ON STRUCTURE OF SEMIGROUPS OF CENTERED UPFAMILIES ON GROUPS

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The through study of various extensions of semigroups was started in [9] and continued in [1]-[8], [10]-[13]. The largest among these extensions is the semigroup $\upsilon(S)$ of all upfamilies on a semigroup S. A family $\mathcal{M}$ of nonempty subsets of a set X is called an upfamily if for each set $A \in \mathcal{M}$ any subset $B \supset A$ of X belongs to $\mathcal{M}$. Each family $\mathcal{B}$ of nonempty subsets of X generates the upfamily $\langle B \subset X : B \in \mathcal{B} \rangle = \{A \subset X : \exists B \in \mathcal{B}(B \subset A)\}$. A family $\mathcal{F}$ of non-empty subsets of a set X that is closed under taking supersets and finite intersections is called a *filter*. A filter $\mathcal{U}$ is called an *ultrafilter* if $\mathcal{U} = \mathcal{F}$ for any filter $\mathcal{F}$ containing $\mathcal{U}$. The family $\beta(X)$ of all ultrafilters on a set X is called the *Stone-Čech compactification* of X, see [14]. An ultrafilter $\langle\{x\}\rangle$, generated by a singleton $\{x\}$, $x \in X$, is called *principal*. Identifying each point $x \in X$