We investigate pairs of Fourier quasicrystals μ_1, μ_2 with discrete supports Λ_1, Λ_2 such that the set of differences $\Lambda_1 - \Lambda_2$ is also discrete. We show that the conditions "$\min_j \inf_{x\in\Lambda_j} |\mu_j(x)| > 0$" and "variations of $\hat{\mu}_j$ are uniformly bounded in any ball of radius 1" imply the supports of both measures are subsets of a unique pure crystal. Noté that here we need not the discreteness of spectra of measures. In the case $\Lambda_1 = \Lambda_2$ we get new conditions for support to be a pure crystal.

J.C.Lagarias (2000) conjectured that if μ is a positive measure with a uniformly discrete support and spectrum, then the support of μ is a subset of a pure crystal. The conjecture was proved by N.Lev and A.Olevski (2015). They also proved the corresponding result for any complex measure on the real axis with uniformly discrete support and spectrum.

We construct the singed measure μ on $\mathbf{R}^2$ such that its support and spectrum are uniformly discrete and simultaneously are not subsets of pure crystals. Nevertheless, their supports are unions of two noncommensurable lattices.

The result agrees with A.Cordoba's one (1989). Namely, a uniformly discrete support Λ of any Fourier quasicrystal μ with the property "the set $\{\mu(x) : x \in \Lambda\}$ is finite" is a finite union of translates of several full-rank lattices (maybe noncommensurable).

We replace the above condition by "$F(\mu(x)) = 0$ for all $x \in \Lambda$" for $F(z) = \sum c_{k,m} z^k \bar{z}^m$ to be any convergent series with $c_{0,0} \neq 0$.

ON STRUCTURE OF SEMIGROUPS OF CENTERED UPFAMILIES ON GROUPS

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The through study of various extensions of semigroups was started in [9] and continued in [1]-[8], [10]-[13]. The largest among these extensions is the semigroup $v(S)$ of all upfamilies on a semigroup S. A family $\mathcal{M}$ of nonempty subsets of a set X is called an upfamily if for each set $A \in \mathcal{M}$ any subset $B \supset A$ of X belongs to $\mathcal{M}$. Each family $\mathcal{B}$ of nonempty subsets of X generates the upfamily $\langle B \subset X : B \in \mathcal{B}\rangle = \{A \subset X : \exists B \in \mathcal{B}(B \subset A)\}$. A family $\mathcal{F}$ of non-empty subsets of a set X that is closed under taking supersets and finite intersections is called a *filter*. A filter $\mathcal{U}$ is called an *ultrafilter* if $\mathcal{U} = \mathcal{F}$ for any filter $\mathcal{F}$ containing $\mathcal{U}$. The family $\beta(X)$ of all ultrafilters on a set X is called the *Stone-Čech compactification* of X, see [14]. An ultrafilter $\langle\{x\}\rangle$, generated by a singleton $\{x\}$, $x \in X$, is called *principal*. Identifying each point $x \in X$

with the principal ultrafilter $\langle\{x\}\rangle$ we obtain the inclusions $X \subset \beta(X) \subset \upsilon(X)$. It was shown in [9] that any associative binary operation $* : S \times S \to S$ can be extended to an associative binary operation $\circ : \upsilon(S) \times \upsilon(S) \to \upsilon(S)$ by the formula $\mathcal{L}\circ\mathcal{M} = \left\langle \bigcup_{a\in L} a * M_a : L \in \mathcal{L},\ \{M_a\}_{a\in L} \subset \mathcal{M} \right\rangle$ for upfamilies $\mathcal{L}, \mathcal{M} \in \upsilon(S)$. In this case the Stone-Čech compactification $\beta(S)$ is a subsemigroup of the semigroup $\upsilon(S)$. The semigroup $\upsilon(S)$ contains many other important extensions of S. In particular, it contains the semigroup $N_{<\omega}(S)$ of centered upfamilies. An upfamily $\mathcal{L} \in \upsilon(S)$ is called *centered* if $\bigcap \mathcal{F} \neq \varnothing$ for any finite subfamily $\mathcal{F} \subset \mathcal{L}$.

Given a group G we we shall discuss the algebraic structure of the extension $N_{<\omega}(G)$ of G. We describe right and left zeros, idempotents, the minimal ideal, left cancelable and right cancelable elements of the semigroup $N_{<\omega}(G)$ of centered upfamilies and characterize groups G whose extensions $N_{<\omega}(G)$ are commutative.

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ON SUFFICIENT CONDITIONS FOR A POLYNOMIAL TO BE SIGN-INDEPENDENTLY HYPERBOLIC OR TO HAVE REAL SEPARATED ZEROS

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The well-known Hutchinson's theorem states that if P be a polynomial with positive coefficients, $P(x) = \sum_{k=0}^{n} a_k x^k$, and $\frac{a_{k-1}^2}{a_{k-2}a_k} \geqslant 4$ for $k = 2, 3, \ldots, n$, then all the zeros of P are real. We obtain sufficient conditions for a real polynomial to be a sign-independently hyperbolic polynomial or to have real separated roots in the style of Hutchinson's theorem.

Joint work with Anna Vishnyakova.

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WIMAN'S INEQUALITY FOR ANALYTIC FUNCTIONS IN $\mathbb{D} \times \mathbb{C}$ WITH RAPIDLY OSCILLATING COEFFICIENTS

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By $\mathcal{A}^2$ we denote the class of analytic functions $f\colon \mathbb{D} \times \mathbb{C} \to \mathbb{C}$, $\mathbb{D} = \{\tau \in \mathbb{C}\colon |\tau| < 1\}$ of the form $f(z) = f(z_1, z_2) = \sum_{n+m=0}^{+\infty} a_{nm} z_1^n z_2^m$, $z = (z_1, z_2)$. For $r = (r_1, r_2) \in T := [0, 1) \times (0, +\infty)$ and $f \in \mathcal{A}^2$ denote

$$\Delta_r = \{t = (t_1, t_2) \in T\colon t_1 \geqslant r_1, t_2 \geqslant r_2\},$$